Name _____

UT EID _____

Discussion Section (8am, 3pm, or 4pm) _____

M408D - 57150, 57155, 57160
Final Exam - Fall 2009 - M. Henry

A Few Notes:

1. Write your name and UT EID in the top left corner of each page. Write the time of your discussion section in the top right corner of each page.
2. Read through all of the problems before you begin and note the point values of each of the parts involved.
3. **Show your work.**

1. (30pts total, 3pts each) Determine whether the statement is true or false. Indicate your answer with a **T** or **F**. You do not need to justify your answer.

1. _____ $\lim_{x\to\pi^-} \frac{\sin x}{1-\cos x} = -\infty$.
2. _____ The two planes defined by $6x + 7y - 2z = 10$ and $12x + 14y - 4z = 60$ are parallel.
3. _____ If the sequence $\{s_n\}$ of partial sums of the series $\sum_{n=0}^{\infty} a_n$ is monotonic and bounded, then $\sum_{n=0}^{\infty} a_n$ is convergent.
4. _____ The gradient vector $\nabla f(x_0, y_0)$ gives the direction of the fastest increase of f at the point (x_0, y_0).
5. _____ The alternating harmonic series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n}$ is convergent.
6. _____ If the sum of $\sum_{i=1}^{\infty} a_i$ is s and $s_n = \sum_{i=1}^{n} a_i$, then $|s - s_n| \leq a_{n+1}$.
7. _____ The volume of the solid under the plane $z = x + 2y$ and above the region in the xy-plane bounded by $y = 0$, $y = x^2$, and $x = 1$ is computed by $\int_0^1 \int_0^{x^2} x + 2y \; dy \; dx$.
8. _____ If the power series $f(x) = 1 - \frac{2}{3}x + \frac{3}{2}x^2 + \ldots$ converges for all values of x, then $f'(0) = 3$.
9. _____ If the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is 2, then the radius of convergence of $\sum_{n=0}^{\infty} c_n (x^2)^n$ is 4.
10. _____ If $f(x, y) \to L$ as $(x, y) \to (a, b)$ along every straight line through (a, b), then $\lim_{(x,y)\to(a,b)} f(x, y) = L$.

2. (10pts)

1. Estimate the volume of the solid that lies above the square $R = [0, 4] \times [0, 4]$ and below the elliptic paraboloid $f(x, y) = 68 - 2x^2 - 2y^2$ by dividing R into four equal squares and using the Midpoint rule. Circle the correct answer below. You do not need to justify your answer.

   (a) 836 (b) 778 (c) 768 (d) 192 (e) 762

2. Compute the exact volume of the solid in part 1. Circle the correct answer below. You do not need to justify your answer.

   (a) $\frac{2240}{3}$ (b) 272 (c) $\frac{176}{3}$ (d) $\frac{2752}{3}$ (e) 760

3. (6pts) Find the values of x such that the vectors $< 3, 2, x >$ and $< 2x, 4, x >$ are orthogonal. Circle the correct answer below. You do not need to justify your answer.

(a) $x = -1$, $x = 2$

(b) $x = -2$, $x = -4$

(c) $x = -2$, $x = 4$

(d) $x = 2$, $x = -4$

(e) $x = -4$, $x = 4$

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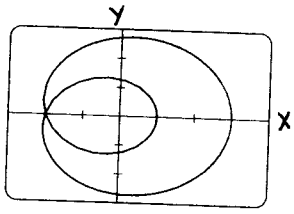
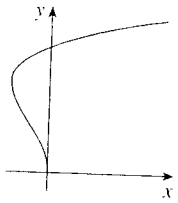
4. (6pts) Match each parametric or polar equation with its graph. Write the letter of the equation under the appropriate picture. You do not need to justify your answer.

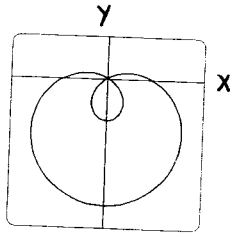
(a) $x = \cos(t) + 2\cos(2t)$, $y = \sin(t) + 2\sin(2t)$

(b) $r = 1 - 2\sin(\theta)$

(c) $x = t^4 - t + 1$, $y = t^2$







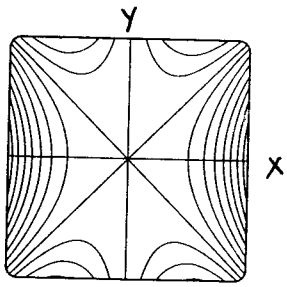
5. (12pts) Match each equation with its graph or contour map. Write the letter of the equation under the appropriate picture. You do not need to justify your answer.

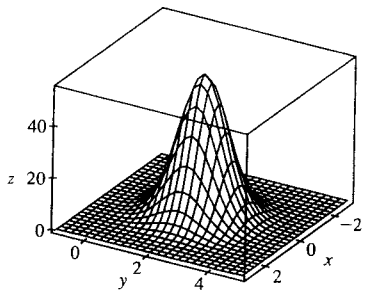
(a) $f(x,y) = xy^2 - x^3$

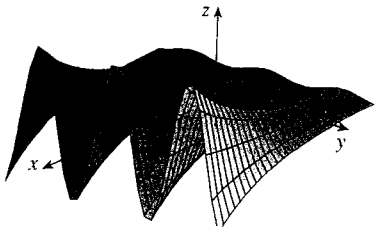
(b) $f(x,y) = xy^3 - yx^3$

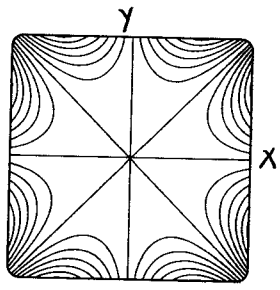
(c) $f(x,y) = e^{4y-x^2-y^2}$

(d) $f(x,y) = e^x\cos(y)$









6. (6pts)

1. Find the domain of the vector function $\mathbf{r}(t) = < t^2, \sqrt{t-1}, \sqrt{9-t} >$

2. Compute $\mathbf{r}'(2)$.

7. (5pts) Determine whether $\int_{-\infty}^{0} xe^x \, dx$ is convergent or divergent. If it is convergent, evaluate it.

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8. (4pts) Sketch the curve given by the polar equation $r = -\theta$, where $0 \leq \theta \leq 2\pi$.

9. (7pts) Find parametric equations for the line of intersection of the planes $x + y + z = 1$ and $x + 2y + 2z = 1$.

11. (10pts) Determine whether each series is absolutely convergent, conditionally convergent or divergent. Justify your answer.

1. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+2}$

2. $\sum_{n=0}^{\infty} \frac{n^3 2^n}{n!}$

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12. (10pts) Find the interval of convergence of the power series $\sum_{n=0}^{\infty} 3^n x^n$.

10. (8pts) Find a nonzero vector orthogonal to the plane through the points $P(3,0,6)$, $Q(2,1,5)$, and $R(-1,3,4)$. Find the area of the triangle PQR.

13. (12pts) Find a power series in x for $\int \frac{1}{1+x^2} dx$. What is the radius of convergence? Justify your answer.

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14. (12pts) Let $f(x, y) = x^4 - x^2y^2 + y^2$. Find all of the critical points of f and classify them as either local minimums, maximums or saddle points.

15. (12pts) Suppose a particle P in the xy-plane is moving around the ellipse $x^2 + \frac{y^2}{4} = 1$. Use Lagrange multipliers to find the point on the ellipse at which P is furthest from the point $Q\left(0, \frac{1}{2}\right)$. Also find the point on the ellipse at which P is closest to the point Q.